Electromagnetic small-scale modeling of composite panels

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Abstract

We are interested in non-destructive electromagnetic characterization of disorganized periodic composite materials composed of a multi-layer infinite plate with a periodic set of circular cylindrical fibers in each layer. The work presented is the preliminary analysis of the scattering of a single-layer plate. An approach based on the multipole method and plane-wave expansion is proposed for obtaining the field representations in all regions of space and forming the formulas for the calculation of reflection and transmission coefficients. To confirm applicability and accuracy of the proposed method, various numerical examples are given when the plate is illuminated by a plane wave.

1. Introduction

The contribution is about the electromagnetic modeling of multi-layer periodic structures, as the fiber composites used in aeronautic and automotive parts. The structure could be treated as a pile of plates one over the other, each made of a regular linear arrangement of long cylinders with same circular sections and all of them oriented into the same direction. The angle between the cylinders of different layers could be any one in the ranges from 0^o to 90^o.

At small scale (small enough local wavelength or skin depth vs. geometry, referring to [1] for large scale), the assumption is that each cell contains one circular cylinder with due repetition, the orientation of the cylinders changing from one slab to the next. Since a given slab can be seen as infinitely periodic if for large lateral extent, it behaves like an infinite array, prone to Floquet-related modeling techniques, to be weighted in vs. the limited spatial extent of the probing fields generated by most NdT probes (coils, dipoles, etc.), which means a finite number of cylinders effectively interacting.

As a first level of modeling, only one layer is considered. A periodic array of parallel circular cylinders is embedded inside a slab. The background materials of the slab (matrix) are different from the materials used for the fibers (reinforcements). The contrast between the electrical parameters of matrix and reinforcements could be very high, such as with carbon-fiber-reinforced polymer, or quite low, such as with glass-fiber-reinforced polymer. Electromagnetic wave propagation in both low-contrast and high-contrast composites was mainly studied in order to deal with the wave reflection and transmission problems related to the mentioned periodic inhomogeneous structures.

The method used to calculate reflection and transmission coefficients is a combination of a multipole method and plane-wave expansion, borrowed from pioneering poro-acoustics and elasticity (infinite extent case) [2] and photonic (limited extent case) [3] analyses. It is an efficient and accurate computation method for periodic structures like ours.

2. Theoretical analysis

The structure under scrutiny is sketched in Figure 1. A set of cylinders parallel to each other and directed in the y direction is embedded in a planar plate infinite in both x and y directions with interfaces Γ_a $(z = a)$ and Γ_b $(z = b)$. The cylinders are arranged periodically in the x direction with period d. Each is of radius c. So, each cell, has a width of d and height of $L = a - b$, the reference (unit) cell being displayed at the center. The space is divided into subspaces $R_0^{\pm}$, R_1 and R_2. All materials are linear isotropic, possibly lossy (save the upper half-space), with ε_j and μ_j, $j = 0, 1, 2$, as permittivities and permeabilities. The TM-polarized incident plane wave with plane of incidence $x - z$ and obliquely impinging with θ_i angle upon the plate has electric field $\vec{E}_{inc} = \hat{y}E_{inc}e^{i(k_x^i x - k_z^i (z-a))}$ with implied time-harmonic dependence $e^{i\omega t}$. $\vec{k}^i$ is the wave number vector, with absolute value k^i, and $k_x^i = k^i \sin\theta^i$, $k_z^i = k^i \cos\theta^i$. The investigation is also suitable to TE-polarized case via application of electromagnetic duality.

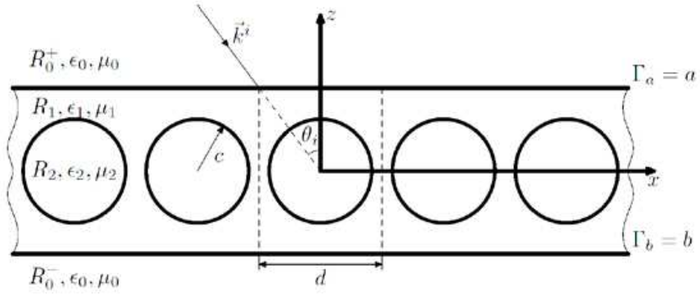

Figure 1 The sketch of the structure considered

The particular feature of the problem is the transverse periodicity of the inclusions in region R_2. According to Floquet theorem, this periodicity and the plane wave nature yield the well-known relation $E_{jy}(x + d, z) = E_{jy}(x, z)e^{i\alpha_0 d}$, where $E_{jy}(x, z)$ are the fields in $R_0^{\pm}$, R_1 and R_2, denoting fields in R_0^+ and R_0^- as $E_{0y}^+(x, z)$ and $E_{0y}^-(x, z)$. The field in R_0^+

and R_0^- can be plane-wave expanded as $E_{0y}^+(x,z) = \sum_{p\in\mathbb{Z}}(E_{inc}e^{-i\beta_{0p}(z-a)}\delta_{0p} + R_p e^{i\beta_{0p}(z-a)})e^{i\alpha_p x}$ and $E_{0y}^-(x,z) = \sum_{p\in\mathbb{Z}} T_p e^{-i\beta_{0p}(z-a)}e^{i\alpha_p x}$, where R_p and T_p are the reflection and transmission coefficients of the plane wave indexed by p, δ_{0p} the Kronecker symbol. $\alpha_p = \alpha_0 + 2p\pi/d$, and $\beta_{jp} = \sqrt{k_j^2 - \alpha_p^2}$. In R_1, the field could be written as $E_{1y}^{\pm}(x,z) = \sum_{p\in\mathbb{Z}}\left(f_p^- e^{-i\beta_{1p}z} + f_p^+ e^{i\beta_{1p}z}\right)e^{i\alpha_p x} + \sum_{p\in\mathbb{Z}}\sum_{l\in\mathbb{Z}} B_l K_{pl}^{\pm} e^{i(\alpha_p x \pm \beta_{1p} z)}$, with $K_{pl}^{\pm} = 2(-i)^l e^{\pm il\theta_p}/d\beta_{1p}$ [4][5]. The magnetic field components readily follow. Then, using the boundary conditions on interfaces Γ_a ($z = a$) and Γ_b ($z = b$), we get the solution for R_p and T_p as

$$\begin{cases} R_p = -\dfrac{1}{D_p}\left[2i\sin(\beta_{1p}L)(h_p^2-1)E_{inc}\delta_{p0} + \displaystyle\sum_{l\in\mathbb{Z}}\dfrac{8(-i)^l h_p B_l}{d\beta_{1p}}(h_p\cos(l\theta_p+\beta_{1p}b) + i\sin(l\theta_p+\beta_{1p}b))\right] \\ T_p = \dfrac{1}{D_p}\left[4h_p E_{inc}\delta_{p0} + \displaystyle\sum_{l\in\mathbb{Z}}\dfrac{8(-i)^l h_p B_l}{d\beta_{1p}}[i\sin(l\theta_p+\beta_{1p}a) - h_p\cos(l\theta_p+\beta_{1p}a)]\right] \end{cases}$$

with $D_p = 2i\sin(\beta_{1p}L)\left(h_p^2+1\right) - h_p\cos(\beta_{1p}L)$, $h_p = \mu_0\beta_{1p}/\mu_0\beta_{1p}$. It is obvious that the transmission and reflection coefficients are related to the coefficients B_l which are calculated with multipole method[4][5][6].

A class of Schlömilch series (lattice sums) arises naturally here. In our 1-D periodicity in a 2-D scattering case, they are defined as $S_l = \sum_{n=1}^{+\infty} H_l^{(1)}(k_1 nd)[e^{i\alpha_0 nd} + (-1)^l e^{-i\alpha_0 nd}]$, wherein $H_l^{(1)}(x)$ is the first-kind Hankel function of lth order, $\alpha_0 = k_0\sin\theta^i$. This representation of the lattice sums converges very slowly for numerical computation. A large literature exists for accelerating the convergence [7]-[11]. In our work, one of the most recent methods [11] is used. The series is transformed into a new expression with elementary functions. It allows the lattice sums be computed accurately and efficiently. For $f = 5$ MHz at normal incidence with the parameters of the glass-fiber-reinforced composites $L = d = 0.1$mm, good results with relative error less than 10^{-6} can be achieved by the method presented in [11] in only 0.0286s on a computer with Intel(R) core(TM) i7-3520 CPU and 8G memory. But if one directly calculates the lattice sum, the convergence of the series could be quite slow and the computational cost is high.

3. Numerical results

First, our approach is validated by comparing with the results in [12] for both dielectric and perfectly conducting cylinders (Figure 2), and they match very well. The investigation for carbon-fiber and glass-fiber composite materials has been completed for normal incident TE/TM waves. For carbon-fiber, the frequency considered is in the range from 1 MHz to 1 GHz. For glass fiber, the frequency band ranges from 10 GHz to 60 GHz. Corresponding results are shown in the poster.

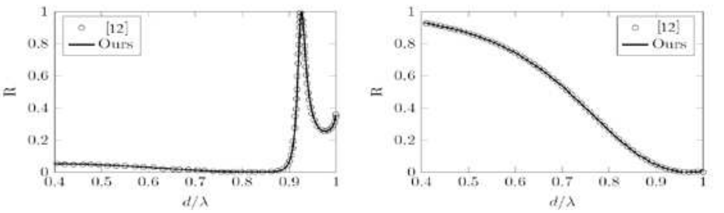

Figure 2. Numerical results for (a) dielectric ($\varepsilon = 2\varepsilon_0$, $c = 0.15$d) and (b) perfect conducting ($c = 0.3$d) cylinders with air matrix. d=1mm.

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